Relations between the $K_{\ell 3}$ and $\tau \to K\pi\nu_\tau$ decays

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Abstract

We investigate the relations between the $K_{\ell 3}$ and $\tau \to K\pi\nu_\tau$ decays using the meson dominance approach. First, the experimental branching fractions (BF) for $K^{\pm}_{e3}$ and K^0_{e3} are used to fix two normalization constants (isospin invariance is not assumed). Then, the BF of $\tau^- \to K^*(892)^-\nu_\tau$ is calculated in agreement with experiment. We further argue that the nonzero value of the slope parameter λ_0 of the $K^{\pm}_{\mu 3}$ and $K^0_{\mu 3}$ form factors $f_0(t)$ implies the existence of the $\tau^- \to K^*_0(1430)^-\nu_\tau$ decay. We calculate its BF, together with BF's of the $K^{\pm}_{\mu 3}$, $K^0_{\mu 3}$, $\tau^- \to K^-\pi^0\nu_\tau$, and $\tau^- \to \bar{K}^0\pi^-\nu_\tau$ decays, as a function of the λ_0 parameter. At some value of λ_0, different for charged and neutral kaons, calculated BF's seem to match existing data and a prediction is obtained for the $\tau \to K\pi\nu$ decays going through the $K^*_0(1430)^-$ resonance.

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With a new generation of high statistics and precise data about the $K_{\ell 3}$, *i.e.* $K \to \pi \ell \nu_\ell$, decays coming soon [1,2] it is possible to think about investigating the problems that were not fully resolved in the previous series of experiments, which ended approximately in the early eighties.

One of the as yet undecided issues is that of the value, or even the sign, of the slope λ_0 in the linear parametrization of the form factor f_0, the definition of which we give below. Some $K^\pm_{\mu 3}$ experiments indicated a non-vanishing negative value, some positive.[1] The situation was analysed by the Particle Data Group in 1982 [4] and a recommended value of 0.004 ± 0.007 was chosen. A very recent experiment [5] with its result of 0.062 ± 0.024 influenced the recommended value, which has now become 0.006 ± 0.007 [3].

The situation with the λ_0 parameter in the $K^0_L \to \pi^\pm \mu^\mp \nu$ ($K^0_{\mu 3}$) decay seems to be a little more definite, at least judging from the recommended value of 0.025 ± 0.006 [4,3] and from all the experiments in the period of 1974-1981 agreeing on the positive sign.

In this note we speculate about consequences which may stem from conclusive establishing a nonzero value of λ_0. Its purpose is not to compete with the elaborate calculations of the $K_{\ell 3}$ form factors, see [6–9], or of the kaon production in τ-lepton decays [10,11]. Our aim is to show on a phenomenological basis in a simple and transparent way possible relations between the $K_{\ell 3}$ and $\tau \to K \pi \nu_\tau$ decays. We mainly argue that a nonzero value of the λ_0 parameter of the $K_{\mu 3}$ decays implies a nonzero decay fraction of the $\tau^- \to K^{*-}_0 \nu_\tau$ decay. Judging from our results and the contemporary experimental upper limit, this decay may be observed soon. The tool we are going to use here is the meson dominance hypothesis, see [12] and references therein.

If we believe in the validity of the standard electroweak model in the leptonic sector, we parametrize the matrix element of the $K_{\ell 3}$ decay in the form [13,14]

$$\mathcal{M}_{K_{\ell 3}} = C \left[f_+(t)\ p^\mu + f_-(t)\ q^\mu \right]\ \bar{u} \gamma_\mu (1 - \gamma_5) v\ , \tag{1}$$

where p (q) is the sum (difference) of the four-momenta of the K and π mesons, $t = q^2$, and u and v are appropriately chosen Dirac spinors of outgoing leptons. This relation defines, up to a normalization factor, the $K_{\ell 3}$ form factors $f_+(t)$ and $f_-(t)$. The normalization used most frequently [15,8] is defined by $C = G_F |V_{us}|/2$ for the $K^\pm_{\ell 3}$ and $C = G_F |V_{us}|/\sqrt{2}$ for the $K^0_{\ell 3}$ decays. It is customary to introduce also the form factor [14]

$$f_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2}\ f_-(t)\ , \tag{2}$$

which corresponds to the $J = 0$ state of the $K - \pi$ system, whereas $f_+(t)$ to its $J = 1$ state. After integrating over angular variables, the differential decay rate in t, which has also a meaning of the invariant mass squared of the $\ell \nu$ system, comes out as

$$\begin{aligned} \frac{d\Gamma_{K_{\ell 3}}}{dt} &= \frac{C^2}{3(4\pi m_K)^3} \frac{\left(t - m_\ell^2\right)^2}{t^3}\ \lambda^{1/2}(t, m_K^2, m_\pi^2) \\ &\times \left[\left(2t + m_\ell^2\right) \lambda(t, m_K^2, m_\pi^2)\, |f_+(t)|^2 + 3 m_\ell^2 \left(m_K^2 - m_\pi^2\right)^2 |f_0(t)|^2 \right]\ , \end{aligned} \tag{3}$$

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[1] We refer the reader to [3] for references and more details.

where $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$. The t-dependence of all form factors is usually studied experimentally in linear approximation

$$f(t) = f(0) \left(1 + \lambda \, \frac{t}{m_\pi^2}\right) \; , \tag{4}$$

although such an approximation was shown [16] to be improper, at least for the $f_+(t)$ form factor of the $K_{e3}^{\pm}$ and K_{e3}^0 decays. The authors of [16] found big discrepancies among λ_+'s from different experiments if a linear approximation was used. They clearly demonstrated the existence of a quadratic term in $f_+(t)$ by showing that its inclusion led to better fits.

There is a peculiarity of the present experimental situation, which is worth mentioning. The μ/e universality requires the form factors be equal for the K_{e3} and $K_{\mu3}$ decays. Assuming the validity of (4) we can express the $R = K_{\mu3}/K_{e3}$ branching ratio as a function of two parameters: λ_+ and λ_0. Knowing the experimental values of the latter we can evaluate R and compare it with the experimental ratio. The $K_{\ell3}^{\pm}$ data pass this consistency check without problems, whereas the contemporary recommended values of the $K_{\ell3}^0$ form factor slopes lead to a little lower ratio than the experimental one (0.676 ± 0.009 against 0.701 ± 0.008). To restore the consistency, one has to sacrifice the μ/e universality and allow a higher value of the λ_+ parameter in the $K_{\mu3}^0$ decay.

A remark is required at the very beginning about our treatment of the $K_{\ell3}$ decays of neutral kaons. We will work with the $K^0 \to \pi^- \ell^+ \nu_\ell$ and $\bar{K}^0 \to \pi^+ \ell^- \bar{\nu}_\ell$ decays, despite the fact that what is really observed are decays of the K_L^0 and K_S^0 mesons. If we ignore a small violation of the CP invariance, then the decay rates of the former two decays are identical and each of them is equal to the decay rate of $K_L^0 \to \pi^\pm \ell^\mp \nu$, where summing is understood over the two final states shown. The same is true for $K_S^0 \to \pi^\pm \ell^\mp \nu$.

The assumption that the $K_{\ell3}$ decay is dominated by the $K^*(892)$ pole, pictorially depicted in Fig. 1a, leads to the following matrix element (see, e.g., [17,12]):

$$\mathcal{M}_{1a} = \frac{G_a}{m_V^2 - t} \left(p^\mu - \frac{m_K^2 - m_\pi^2}{m_V^2} \; q^\mu\right) \bar{u}\gamma_\mu(1-\gamma_5)v \; , \tag{5}$$

where m_V is the $K^{*\pm}(892)$ mass and (dimensionless) G_a collects the coupling constants from all vertices. It includes also the V_{us} element of the Cabibbo-Kobayashi-Maskawa matrix. As the isospin invariance is badly broken in the $K_{\ell3}$ decays, see, *e.g.*, the discussion in [15], we have two independent constants. One for $K_{\ell3}$ decays of $K^\pm$, another for K^0 ($\bar{K}^0$). We do not need the explicit form of G_a's, because we will fix their values from the experimental values of the corresponding K_{e3} decay rates. Nevertheless, in the notation of Ref. [12] we have

$$G_a^{(\pm)} = G_F V_{us} w_{K^*} m_V^2 \frac{g_{K^{*\pm}K^\pm\pi^0}}{g_\rho} \tag{6}$$

and a similar relation for $G_a^{(0)}$. The connection with the standard notation [15,8] is given by $G_a^{(\pm)}/m_V^2 = G_F \, |V_{us}| \, f_+^{K^+\pi^0}(0)/2$. For $G_a^{(0)}$, the factor of 2 is replaced by $\sqrt{2}$.

Let us note that when writing (5) we took the propagator of the K^* resonance in the free-vector-particle form

$$-iG_0^{\mu\nu}(q) = \frac{-g^{\mu\nu} + q^\mu q^\nu / m_V^2}{t - m_V^2 + i\epsilon} \ , \tag{7}$$

where m_V is the mass of the $K^*(892)^-$ resonance, as seen in the hadronic production experiments. The absence of a noninfinitesimal imaginary part in denominator is justified by t being below the threshold of the $K^* \to K\pi$ decay channel. But the actual form of the propagator may differ from (7) even in the subtreshold region. The success in describing the $K_{\ell 3}$ form factors gives an *a posteriori* phenomenological argument in favor of an approximate validity of Eq. (7).

If we fix, for simplicity, the normalization of the form factors by requiring $f_+(0) = 1$, we find the following correspondence of (5) with the quantities entering Eq. (1):

$$\begin{aligned} C &= \frac{G_a}{m_V^2} \ , \\ f_+(t) &= \frac{m_V^2}{m_V^2 - t} \ , \\ f_-(t) &= -\frac{m_K^2 - m_\pi^2}{m_V^2 - t} \ . \end{aligned} \tag{8}$$

We also have

$$f_0(t) = 1 \ . \tag{9}$$

Inserting our C, $f_+(t)$, and $f_0(t)$ to the general formula (3), integrating over t, and comparing our result with the $K_{e3}^\pm$ (K_{e3}^0) decay rate calculated from the experimental values of the $K^\pm$ (K_L^0) lifetime and the $K_{e3}^\pm$ (K_{e3}^0) branching fraction we arrive at ${G_a^{(\pm)}}^2 = (1.037 \pm 0.013) \times 10^{-12}$ and ${G_a^{(0)}}^2 = (1.974 \pm 0.021) \times 10^{-12}$. If the isospin invariance in the $K^*K\pi$ vertex were exact, the ratio of the former to the latter would be equal to 1/2.

Before proceeding further with our form-factor issue let us notice that the same overall coupling constants govern also the decays $\tau^- \to K^-\pi^0\nu_\tau$ and $\tau^- \to \bar{K}^0\pi^-\nu_\tau$ in which the $K\pi$ system is produced via the K^{*-} resonance, see Fig. 2a. Let us first calculate their branching fractions using the G_a's we have just determined. This will test the soundness of our approach and of the approximations made and will give us the confidence for calculations for which the comparison with data is impossible as yet.

The main problem we are faced with when attempting such a calculation is that of the propagators of resonances. We are now above the threshold of the $K\pi$ system, $s > (m_K + m_\pi)^2$, where s is the square of the four-momentum p flowing through the K^* resonance. As a consequence, the propagator acquires an important imaginary part and may differ substantially from the propagator of a free vector particle also in other respects. For example, in [18] it was proposed that the lowest order $W^\pm$ (Z^0) renormalized propagator in the unitary gauge can be obtained, at least in the resonance region, by a simple modification of the free propagator (7). Namely, by replacing the mass squared m_V^2 everywhere in Eq. (7) by $m_V^2 - im_V\Gamma_V$, with Γ_V being the resonance width.[2] For resonances with strong interaction

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[2]For later development and references to alternative approaches to the weak-gauge-bosons propagators see Ref. [19].

such a simple prescription is not justified, as discussed, *e.g.*, in [20]. Nevertheless, if $s = p^2$ is in a close proximity to the resonant mass squared we can write

$$- iG^{\mu\nu}(p) = \frac{-g^{\mu\nu} + \omega(s)p^\mu p^\nu/s}{s - m_V^2 + im_V\Gamma_V(s)} \ , \tag{10}$$

where $\Gamma_V(s)$ is the s-dependent total width of the resonance normalized by $\Gamma(m_V^2) = \Gamma_V$ and $\omega(s)$ is a complex function. It reflects the properties of the one-particle-irreducible bubble and is, in principle, calculable. There are different ways of treating it in practice. For example, when considering the a_1 resonance in the intermediate state, the authors of [20] eliminated its influence by choosing transverse vertices. Alternatively, various choices have been made in the literature. Very popular is the free-particle choice $\omega(s) = s/m_V^2$, recently used, *e.g.*, in Ref. [11]. In experimental analyses a spin-zero propagator is used even where not justified (see discussion in [21]). This corresponds to $\omega(s) = 0$. The same choice was made in [22], where the branching fraction of the $\tau^- \to K^*(892)^-\nu_\tau$ decay was also calculated.

Fortunately, the $K^*(892)$ resonance is relatively narrow ($\Gamma_V \approx 51$ MeV) and we can hope that the systematic error connected with the propagator ambiguity is small. Nevertheless, to assess it we will calculate every quantity of interest twice. Once with $\omega = s/[m_V^2 - im_V\Gamma_V(s)]$, then with $\omega = 0$. This procedure yields an average and an estimate of its systematic error.

The differential rate of the $\tau^- \to K^-\pi^0\nu_\tau$ decay in the mass squared of the $K\pi$ system is given by the formula

$$\begin{aligned} \frac{d\Gamma_{\tau^-\to K^-\pi^0\nu_\tau}}{ds} &= \frac{1}{6(4\pi m_\tau)^3}\frac{(m_\tau^2 - s)^2}{s^3}\ \lambda^{1/2}(s, m_K^2, m_\pi^2) \\ &\times \left[\left(2s + m_\tau^2\right)\lambda(s, m_K^2, m_\pi^2)\,|F_+(s)|^2 + 3m_\tau^2\left(m_K^2 - m_\pi^2\right)^2|F_0(s)|^2\right] \ , \end{aligned} \tag{11}$$

where

$$F_+(s) = \frac{G_a^{(\pm)}}{s - m_V^2 + im_V\Gamma_V(s)} \tag{12}$$

and

$$F_0(s) = \frac{G_a^{(\pm)}\,[1 - \omega(s)]}{s - m_V^2 + im_V\Gamma_V(s)} \ . \tag{13}$$

The presence of $F_0(s)$ in (11) reflects the contribution of the off-mass-shell vector resonance K^* to the $J = 0$ channel. It would disappear if we chose $\omega(s) \equiv 1$, as seen from (13). After integrating (11) and using the experimental value of the τ^- lifetime, we arrive at $B(\tau^- \to K^-\pi^0\nu_\tau) = (3.9 \pm 0.6) \times 10^{-3}$. We proceed similarly to obtain $B(\tau^- \to \bar{K}^0\pi^-\nu_\tau) = (7.1 \pm 1.2) \times 10^{-3}$. After summing these two branching fractions we get

$$B(\tau^- \to K^*(892)^-\nu_\tau) = (1.10 \pm 0.18)\% \ . \tag{14}$$

The experimental value [3] is $(1.28 \pm 0.08)\%$.

Let us now return to the form factors. The salient feature of the one-vector-meson dominance model is the constant $K_{\ell 3}$ form factor f_0, which implies a vanishing parameter λ_0 defined in (4). There are at least two ways to accommodate a nonvanishing value of λ_0 in the meson dominance approach.

One possibility is to add more strange vector resonances. The case of two vector resonances was considered already in [17]. In addition to the well established $K^*(892)$ it was $K^*(730)$, which was abandoned later on. But the formulas of [17] are general, and could be used for inclusion of $K^*(1410)$ as well.

Another way of modifying the meson dominance approach to the $K_{\ell 3}$ decay is to include the scalar resonance $K_0^*(1430)$. The advantage of this approach is that, as we will see, it does not modify the $f_+(t)$ form factor, which seems to be well described already with the $K^*(892)$ alone. The modification influences only the $f_-(t)$ and, consequently, the $f_0(t)$ form factors. Already the authors of [23] discussed this possibility, but at that time there was no known K-π resonance with spin zero.

To calculate the contribution to the $K_{\ell 3}$ matrix element from the Feynman diagram with the $K_0^*(1430)^-$ in the intermediate state (Fig. 1b), let us first define the weak decay constant of the K_0^{*-}. As usual, it can be done by means of the matrix element of the vector part of the strangeness-changing quark current

$$\langle 0|\bar{u}(0)\gamma^\mu s(0)|p\rangle_{K_0^{*-}} = i\ f_{K_0^{*-}}\ p^\mu\ . \tag{15}$$

Then, the diagram in Fig. 1b yields

$$\mathcal{M}_{1b} = \frac{G_b}{m_S^2 - t}\ q^\mu \bar{u}\gamma_\mu(1-\gamma_5)v\ , \tag{16}$$

where m_S is the $K_0^*(1430)^-$ mass and

$$G_b = \frac{G_F}{\sqrt{2}} V_{us} f_{K_0^{*-}} g_{K_0^{*-}K^-\pi^0}\ . \tag{17}$$

Because (16) does not contain P^μ, the constant C and the form factor $f_+(t)$, as shown in (8), will not change after adding (16) to (5). New $f_-(t)$ and $f_0(t)$ become

$$\begin{aligned} f_-(t) &= -\frac{m_K^2 - m_\pi^2}{m_V^2 - t} + \frac{G_b}{G_a}\frac{m_V^2}{m_S^2 - t}\ , \\ f_0(t) &= 1 + \frac{G_b}{G_a}\frac{m_V^2}{m_K^2 - m_\pi^2}\frac{t}{m_S^2 - t}\ . \end{aligned} \tag{18}$$

The parameter λ_0 now acquires the value

$$\lambda_0 = \frac{G_b}{G_a}\frac{m_V^2}{m_S^2}\frac{m_\pi^2}{(m_K^2 - m_\pi^2)}\ . \tag{19}$$

We see that the nonzero weak decay constant of K_0^{*-} leads to deviation of the λ_0 parameter from zero. But to check whether a nonvanishing value of λ_0 is really caused by a K_0^{*-} in the intermediate state of the $K_{\ell 3}$ decay, we must look for other consequences of the weak

interaction of K_0^{*-} and their consistency with the $K_{\ell 3}$ decay phenomenology. The most obvious candidate for such a program is the decay of τ^- lepton to neutrino and K_0^{*-}. Or, to be more precise, to the $K^-\pi^0$ system which originates from the strong decay of K_0^{*-}.

When calculating the branching fraction of the $\tau^- \to K^-\pi^0\nu_\tau$ and $\tau^- \to \bar{K}^0\pi^-\nu_\tau$ decays, we should include the possible interference between the $K^*(892)^-$ and $K_0^*(1430)^-$ channels, *i.e.*, add coherently the diagrams (a) and (b) shown in Fig. 2. The resulting differential decay rate formula for $\tau^- \to K^-\pi^0\nu_\tau$ coincides with Eq. (11). Function $F_+(s)$ is again given by (13) because the scalar resonance cannot contribute to the $J = 1$ channel, but

$$F_0(s) = \frac{G_a^{(\pm)}\,[1-\omega(s)]}{s - m_V^2 + i m_V \Gamma_V(s)} + \frac{G_b^{(\pm)}}{m_K^2 - m_\pi^2}\frac{s}{s - m_S^2 + i m_S \Gamma_S(s)}\ . \tag{20}$$

The changes needed to get a formula for the same quantity in $\tau^- \to \bar{K}^0\pi^-\nu_\tau$ are obvious.

Now we have all necessary formulas and constants prepared and can calculate the quantities of interest for various values of the slope parameter λ_0. The results are shown in Tab. I for the charged kaons, in Tab. II for the neutral kaons.

Inspecting Tab. I we see that to get simultaneously the correct branching fraction of both $K_{\mu 3}^{\pm}$ and $\tau^- \to K^-\pi^0\nu_\tau$ decays, we need to pick $\lambda_0 \approx 0.020$. This is higher than the present recommended value $(6 \pm 7) \times 10^{-3}$. But with eyes on the recent experiment [5] with its 0.062 ± 0.024, we do not consider disastrous the dicrepancy of our value of λ_0 with the recommended one. Our value also agrees with $\lambda_0 = 0.019$ obtained on the basis of the Callan-Treiman relation [24], see [8]. With reference to the experiment [5] it should be said that $\lambda > 0.04$ contradicts the estimate of the upper limit for the non-$K^*(892)^-$ $K^-\pi^0$ production in τ^- decays. On the basis of $\lambda_0 \approx 0.020$ we expect the branching fraction for producing the $K^-\pi^0$ system in τ^- decays via the scalar $K_0^*(1430)^-$ resonance to be $\approx 2 \times 10^{-4}$.

Similar analysis of numbers in Tab. II points to a λ_0 for the $K_{\mu 3}^0$ decay somewhere around 0.030, which is in agreement with the recommended value [3], but higher than in the previous case. The higher value is required by the $K_{\mu 3}^0$ branching fraction. As a consequence, also the branching fraction of the $\bar{K}^0\pi^-$ production from the $\tau^- \to K_0^*(1430)^-\nu_\tau$ decay, $\approx 8 \times 10^{-4}$, is higher than would correspond to $K^-\pi^0$ and isospin symmetry.

On the basis of our estimates we expect the branching fraction of the $\tau^- \to K_0^*(1430)^-\nu_\tau$ decay to be around 0.1%.

In Fig. 3 we show the mass spectrum of the $\bar{K}^0\pi^-$ system produced in the $\tau^- \to \bar{K}^0\pi^-\nu_\tau$ decays assuming $\lambda_0 = 0.030$. We concentrate on the $K_0^*(1430)^-$ mass region to show different contributions to the final yield. The tail of the $K^*(892)^-$ resonance modifies the resonance shape significantly, whereas the interference between the two contributing intermediate states is negligible.

We hope that in the near future the high statistics and precise kaon decay data on one side, and data from the τ-factories [25] on the other, will enable to study the relations between the $K_{\ell 3}$ and $\tau \to K\pi\nu$ decays in more detail.

Finally, we would like to comment the role of the meson dominance model. It is clear that this approach cannot substitute for a more fundamental theory based on first principles. We cannot even say in advance for which processes it will offer a fair description and for which it will fail. But, in our opinion, it has an important role as a heuristic tool. In

the cases where it succeeds, it shows which underlying quark diagrams are most important for understanding the dynamics of the process. Because of the necessity of convoluting the simple pure electroweak diagrams with QCD dynamics in order to form hadrons, the diagrams which seem to be important on the quark level, may finally become unimportant and vice versa. We discussed this aspect in some detail in [12] in connection with the $K^+ \to \pi^+ e^+ e^-$ decay. Also here, the ability of the meson dominance to describe, with the same set of basic parameters, both $K_{\ell 3}$ and $\tau^- \to K\pi\nu$ decays hints that the most important mechanism of destroying strangeness is the quark-antiquark annihilation to the W boson. This picture differs completely from the usual notion in which the non-strange quark is a spectator and proceeds through the process intact, whereas the strange quark converts to a non-strange one by emitting W.

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TABLES

TABLE I. Branching fractions of the $K^{\pm}_{\mu 3}$ and $\tau^- \to K^- \pi^0 \nu_\tau$ decays calculated within the meson dominance approach assuming various values of the $K^{\pm}_{\mu 3}$ parameter λ_0. The recommended experimental values [3] are shown in the last row.

$\lambda_0 \times 10^3$	$B(K^{\pm}_{\mu 3})$ (%)	$B(\tau^- \to K^- \pi^0 \nu_\tau)$ via $K_0^*(1430)^-$	$B(\tau^- \to K^- \pi^0 \nu_\tau) \times 10^3$ total (a)
-10	3.03 ± 0.02	5.3×10^{-5}	3.9 ± 0.6
-5	3.06 ± 0.02	1.3×10^{-5}	3.9 ± 0.6
0	3.10 ± 0.02	0	3.9 ± 0.6
5	3.13 ± 0.02	1.3×10^{-5}	3.9 ± 0.6
10	3.17 ± 0.02	5.3×10^{-5}	4.0 ± 0.6
15	3.21 ± 0.02	1.2×10^{-4}	4.0 ± 0.6
20	3.25 ± 0.02	2.1×10^{-4}	4.1 ± 0.6
25	3.29 ± 0.02	3.3×10^{-4}	4.3 ± 0.6
30	3.33 ± 0.02	4.8×10^{-4}	4.4 ± 0.6
35	3.37 ± 0.02	6.5×10^{-4}	4.6 ± 0.6
40	3.41 ± 0.02	8.4×10^{-4}	4.8 ± 0.6
45	3.45 ± 0.02	1.1×10^{-3}	5.1 ± 0.6
50	3.49 ± 0.02	1.3×10^{-3}	5.3 ± 0.6
55	3.53 ± 0.02	1.6×10^{-3}	5.6 ± 0.6
60	3.58 ± 0.02	1.9×10^{-3}	6.0 ± 0.6
6 ± 7	3.18 ± 0.08	$< 9 \times 10^{-4}$ (b)	5.2 ± 0.5

(a) Total = $K^*(892)^-$ + $K_0^*(1430)^-$ + interference term.
(b) Estimated as a half of $\tau^- \to \pi^- \bar{K}^0 \nu_\tau$, non-$K^*(892)^-$.

TABLE II. Branching fractions of the $K^0_{\mu 3}$ and $\tau^- \to \bar{K}^0\pi^-\nu_\tau$ decays calculated within the meson dominance approach assuming various values of the $K^0_{\mu 3}$ parameter λ_0. The recommended experimental values [3] are shown in the last row.

$\lambda_0 \times 10^3$	$B(K^0_{\mu 3})$ (%)	$B(\tau^- \to \bar{K}^0\pi^-\nu_\tau)$ via $K_0^*(1430)^-$	$B(\tau^- \to \bar{K}^0\pi^-\nu_\tau) \times 10^3$ total (a)
-10	24.39 ± 0.17	8.9×10^{-5}	7.2 ± 1.2
-5	24.65 ± 0.17	2.2×10^{-5}	7.1 ± 1.2
0	24.91 ± 0.17	0	7.1 ± 1.2
5	25.18 ± 0.18	2.2×10^{-5}	7.2 ± 1.2
10	25.46 ± 0.18	8.9×10^{-5}	7.3 ± 1.1
15	25.74 ± 0.18	2.0×10^{-4}	7.4 ± 1.1
20	26.02 ± 0.18	3.6×10^{-4}	7.6 ± 1.1
25	26.31 ± 0.18	5.6×10^{-4}	7.8 ± 1.1
30	26.61 ± 0.19	8.0×10^{-4}	8.1 ± 1.1
35	26.91 ± 0.19	1.1×10^{-3}	8.4 ± 1.1
40	27.21 ± 0.19	1.4×10^{-3}	8.8 ± 1.1
45	27.52 ± 0.19	1.8×10^{-3}	9.2 ± 1.1
50	27.84 ± 0.19	2.2×10^{-3}	9.6 ± 1.1
55	28.15 ± 0.20	2.7×10^{-3}	10.1 ± 1.1
60	28.48 ± 0.20	3.2×10^{-3}	10.7 ± 1.1
25 ± 6	27.17 ± 0.25	$< 1.7 \times 10^{-3}$ (b)	8.3 ± 0.8

(a) Total = $K^*(892)^-$ + $K_0^*(1430)^-$ + interference term.
(b) Non-$K^*(892)^-$ ν_τ.

FIGURES

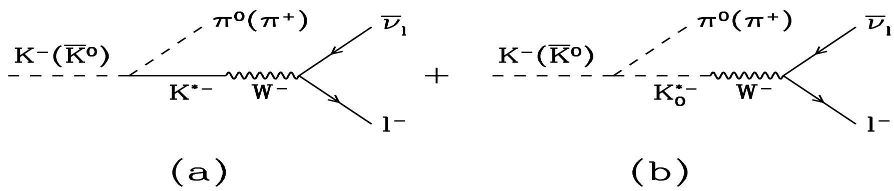

FIG. 1. Feynman diagrams contributing to the matrix element of the $K^- \to \pi^0 \ell^- \bar{\nu}_\ell$ ($\bar{K}^0 \to \pi^+ \ell^- \bar{\nu}_\ell$) decay with (a) the vector resonance $K^*(892)^-$ and (b) the scalar resonance $K_0^*(1430)^-$ in the intermediate state.

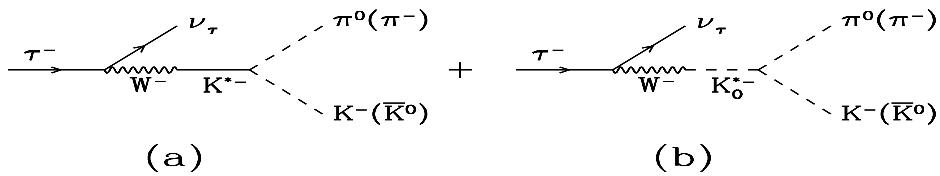

FIG. 2. Feynman diagrams contributing to the matrix element of the $\tau^- \to K^-\pi^0\nu_\tau$ ($\tau^- \to \bar{K}^0\pi^-\nu_\tau$) decay with (a) the vector resonance $K^*(892)^-$ and (b) the scalar resonance $K_0^*(1430)^-$ in the intermediate state.

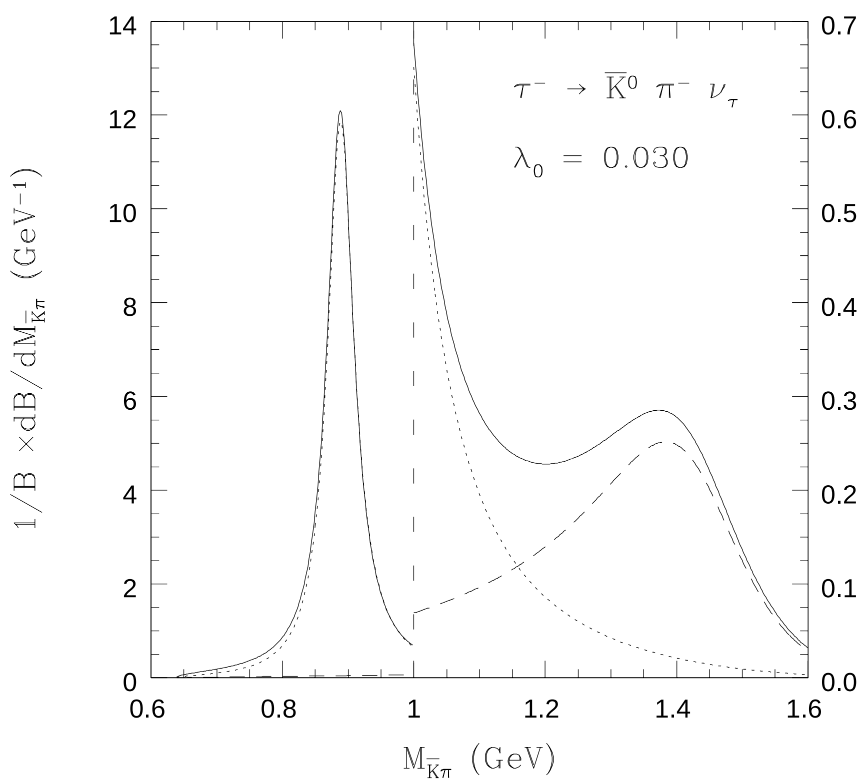

FIG. 3. Mass spectrum of the $\bar{K}\pi$ system produced in the $\tau^- \to \bar{K}^0\pi^-\nu_\tau$ decay with $K^*(892)^-$ and $K_0^*(1430)^-$ in the intermediate state assuming the $K^0_{\mu 3}$ parameter $\lambda_0 = 0.030$. Solid curve: the total branching fraction; dotted curve: $K^*(892)^-$ only; dashed curve: $K_0^*(1430)^-$ only. Notice a different scale for masses above 1 GeV.